Symmetric Encryption via Keyrings and ECC

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ArcticCrypt
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Outline

Motivation—Simplifying Crypto Key Updates

- Keyring (Bag of Words) Model
- Incremental Key Updates
- Keyring Issues

Resilience

- Prior Work—Biometrics, Fuzziness, Quantum
- Resilient Set Vectorization
- Security Analysis

Encrypting with keyrings

- Error-correction
- Keyring encryption details
- Attacks

Discussion

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$\longleftrightarrow$

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CSAIL

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- are **not memorable**, and
- are **updated "all-at-once"** instead of incrementally.

Are there better (non-PK) methods?

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Keyring (Bag of Words) Model

Main idea: Key is a "bag of words" agreed upon by sender and receiver. (Really "set" not "bag" (multiset).)

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- Separate keyring for each sender/receiver pair.
- Sender and receiver have identical (*or nearly identical*) keyrings.
- Maybe 10–100 keywords on a keyring.

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Incremental Key Updates Are Simple

Alice says privately to Bob:

- Let’s add “**garlic**” to our keyring.

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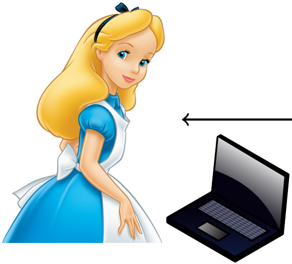
- Let's delete all keywords added in 2015.

Scenario



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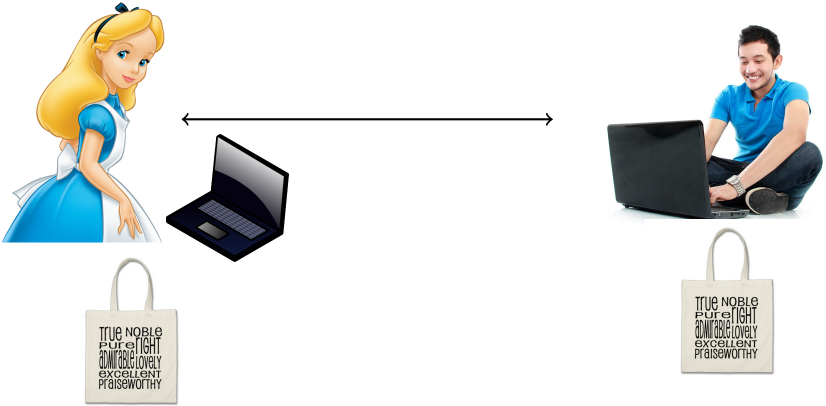
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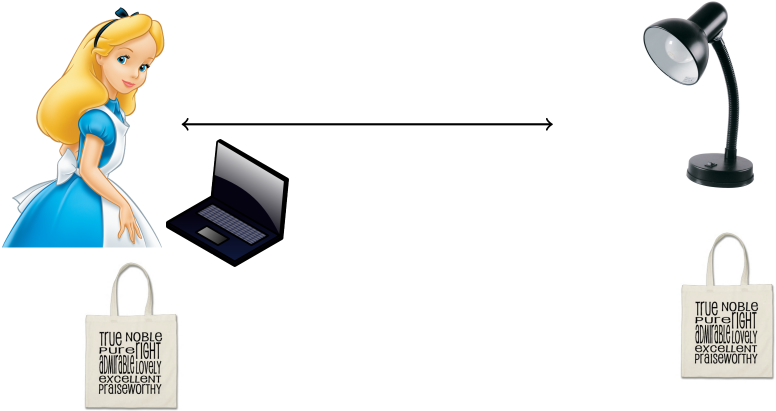

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- **(Keying)** How to use a "bag of words" as a symmetric crypto key?
- **(Security)** How to keep adversary from breaking in and then "tracking" keyring evolution?

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We want that a ciphertext made using keyring A can be decrypted using different keyring B, as long as A and B are “close”.

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We describe a nice way of converting from the first to the second.

Biometrics: Use a fingerprint as key

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Similar to the problem of encrypting a key with a biometric; biometric features $\sim$ keywords.

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- Sahai/Waters 2005 *“Fuzzy IBE”*. Fuzzy PK scheme.

PinSketch[DORS04]

- Uses BCH ECC with algorithms that work efficiently on *sparse* vectors.
- Message transmitted has length δ over $GF(2^\alpha)$, where $2^\alpha \geq |\mathcal{U}|$ and $\mathcal{U}$ is universe of keys, and where δ is upper bound on the size of the set difference $A \oplus B$.
- Allows recipient to reconstruct A.

Quantum Key Distribution

- Bennet Brassard 1984
  “Quantum cryptography: Public key distribution and coin tossing”
  Information reconciliation by public discussion over a classical channel.

Resilient Set Vectorization

A **set vectorizer** ϕ takes as input a set A, an integer n, and a nonce N, and produces as output a uniformly chosen (over the choice of nonce) vector from A^n.

A **resilient set vectorizer** is a set vectorizer with the property that for any two sets A and B with $|A \cap B| = p \cdot |A \cup B|$ (for some p, $0 \le p \le 1$), we have

$$d(\phi(A, n, N), \phi(B, n, N)) \sim n - \mathrm{Bin}(n, p) \ .$$

That is, if a fraction p of $A \cup B$ are shared, then the fraction of positions where $\phi(A, n, N)$ and $\phi(B, n, N)$ agree follows the binomial distribution with mean np.

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- They are given the *same* random nonce N.
- Alice and Bob separately each pick *one* element from their keyrings.
- *What is the maximum probability that they pick the same element, using optimal strategy?*

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Simplest interesting example

$$|A| = 2 \qquad |A \cap B| = 1 \quad |B| = 2 \qquad |\mathcal{U}| = 3$$

CAT DOG RAT

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Alice sees:

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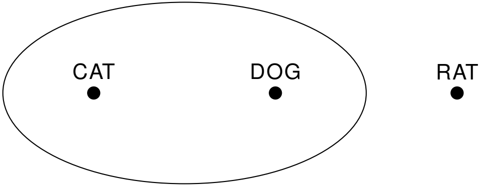

$N = 3762134912$

Should Alice pick CAT or DOG?

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Bob sees:

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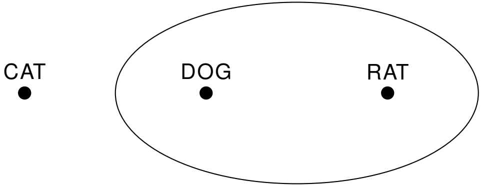

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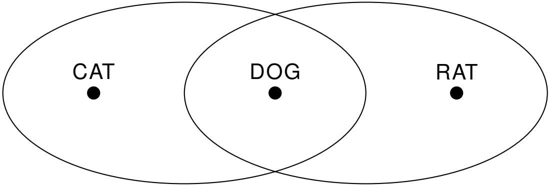

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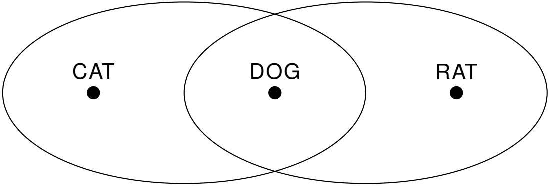

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Agree with prob 1/4? 1/3? 1/2?...

Keyword Matching Game – Random Strategy

- If Alice and Bob make their choices **independently at random**, then they match with probability

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$$|A \cap B| / |A|\,|B| \ .$$

- (Pretty small, especially when A and B are large.)

Keyword Matching Game for $|A \cap B| = 1$

Brute-force searches for optimal strategies (surprisingly) suggested the following

Theorem

When $|A \cap B| = 1$ *and* $A \cup B = \mathcal{U}$ *the optimum match probability is at least*

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But $|A \cap B| = 1$ and $A \cup B = \mathcal{U}$ are unrealistic...

Jaccard Index of Similarity

- The **Jaccard similarity coefficient** $J(A, B)$ measures the similarity of two sets A and B:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} .$$

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- It can be estimated using the **MinHash** method (Broder 1997): Construct n random hash functions mapping elements to real values. Compute the fraction f of them having the same minimum in A as in B. Then

$$E(f) = J(A,B) \; .$$

Keyword Matching Game via MinHash

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Alice and Bob can always win with probability at least $p = J(A, B) = |A \cap B| / |A \cup B|$.

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- Initially, Alice and Bob agree on a random hash function h.
- They each pick their keyword with minimum hash-value.
- They win if one of their shared keywords has the smallest hash value in both sets.

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Conjecture: The MinHash strategy is *optimal* for $|A \cap B| > 1$.

Resilient Set Vectorization (RSV)

Alice iterates the MinHash method (with n random hash functions), to create a **keyword vector**

$$W = \phi(A, n, N) = (W_1, W_2, \ldots, W_n)$$

of some desired length n.

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Let z denote the number of positions in which W and W' agree, and let $p = J(A, B)$. Then (under ROM)

$$z \sim \mathrm{Bin}(n, p),$$

so $E(z) = np$ and $\sigma(z) = \sqrt{np(1-p)}$.

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Suppose further we can arrange things so that the Adversary *can't decrypt* Alice's ciphertext if the number z' of positions of W it knows (or guesses) correctly satisfies

$$z' < n/2 \; .$$

Analysis–for the good guys

- Suppose Alice and Bob have

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- If $z \geq 192$, Bob can decrypt the message.
- Bob fails to decrypt with near-zero probability:

$$\mathrm{Prob}\,(z < 192) = 1.5 \times 10^{-12} \ .$$

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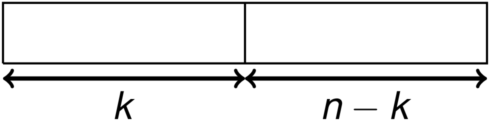
- Adversary's vector $\phi(Q, n, N)$ agrees with Alice's in z' positions.
- If $z' \geq 128$, Adversary can decrypt message.
- But Adversary fails almost certainly, since
$$\text{Prob}\,(z' \geq 128) = 7.5 \times 10^{-18} \ .$$

Error Correction

- An (n, k) Reed-Solomon code has k information symbols and codewords of length n.



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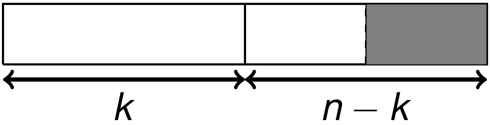

Error Correction

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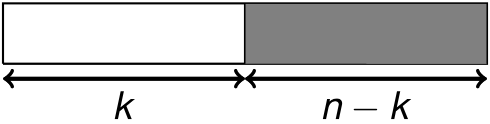
- Bob can efficiently correct up to $(n-k)/2$ errors and always obtain a **unique decoding**.
- With **list decoding** Adversary can efficiently correct up to $(n-k)$ errors (and obtain a small number of possible decodings).



Keyring proposal for encrypting M with keyring A

M

A

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$K_1 \bullet \bullet \bullet K_k$

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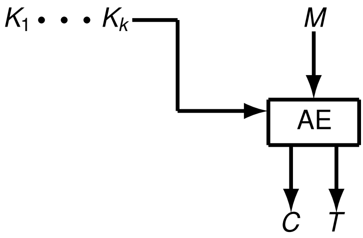

Alice sends C, and T.

Keyring proposal for encrypting *M* with keyring *A*

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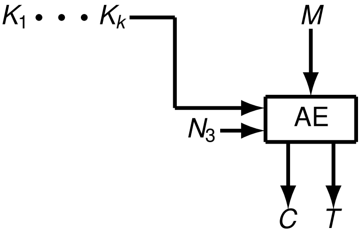

Alice sends (N_3), C, and T.

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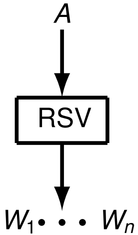

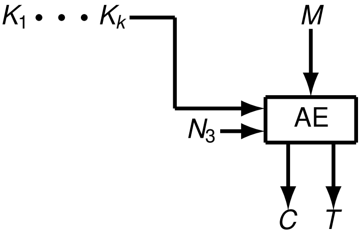

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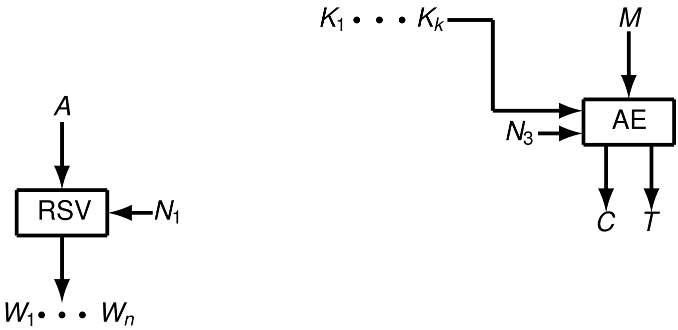

Alice sends (N_1, N_3), C, and T.

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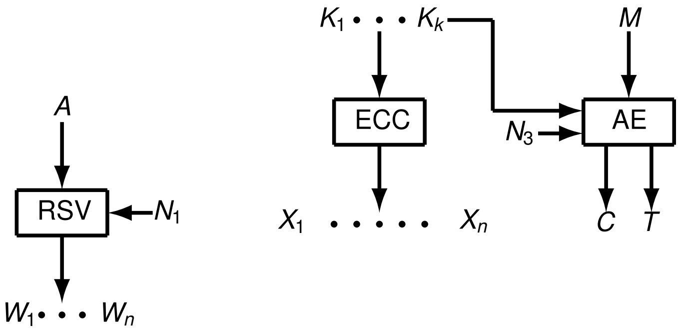

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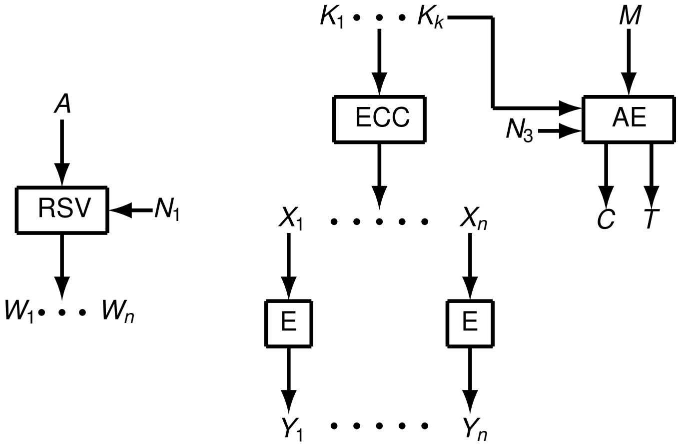
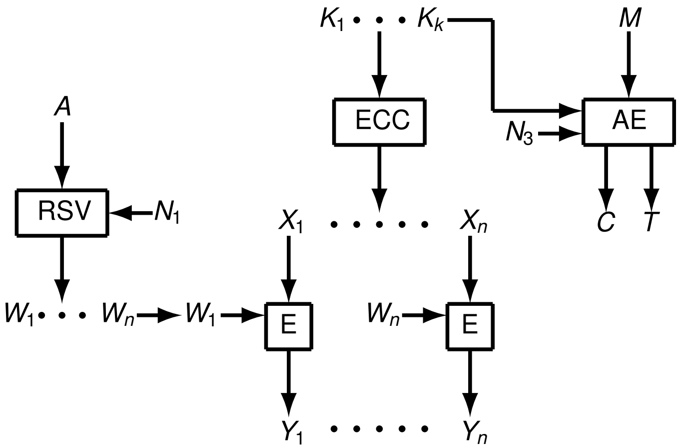


Alice sends (N_1, N_3), Y, C, and T.

Keyring proposal for encrypting M with keyring A



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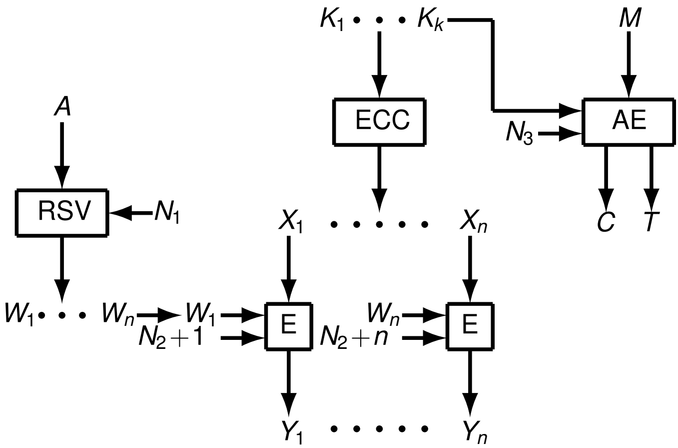

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- Encrypt message M with key K and nonce N_3 using an authenticated encryption method to obtain ciphertext C and authentication tag T.

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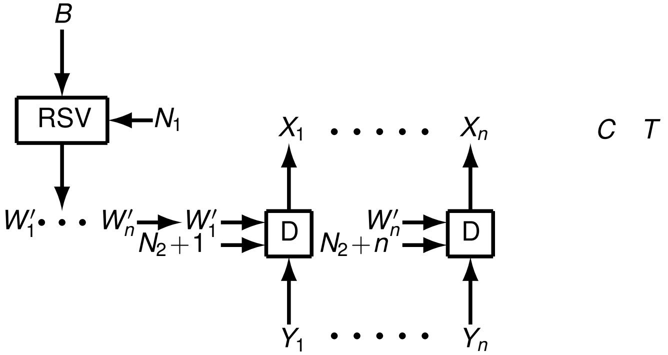

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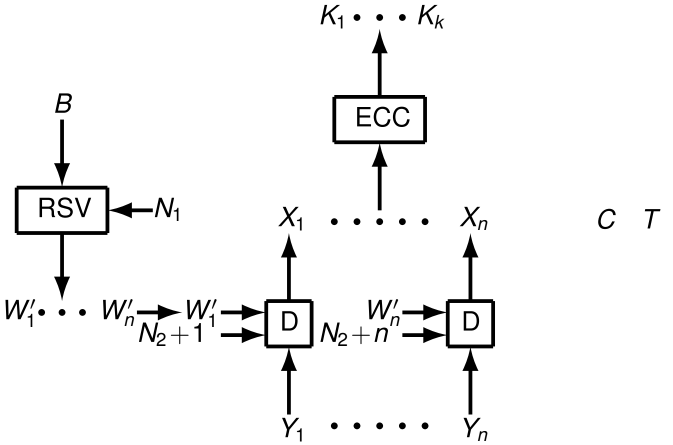

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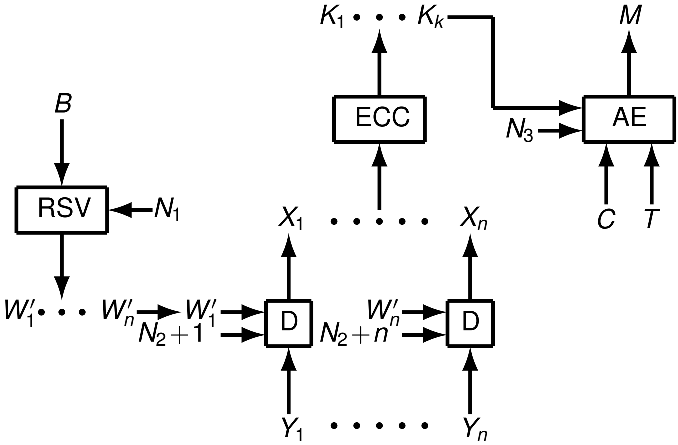

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- Initial keywords may be high-entropy.

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- Make updates large every once in a while!
- Reminiscent of problems of refreshing entropy pool in PRNG. (Ferguson-Schneier-Kohn'10, Dodis-Shamir-StephensDavidowitz-Wich'14).

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- We need to prove that MinHash strategy is optimal (for $|A \cap B| > 1$)!

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- Encrypt M with AEAD instead of AE, where AD includes Y and nonces. Insecure? (AD and K are related.) Proof needed.

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- We send $n = 256$ bytes plus nonces.
- Bob can decode whp if $p - k/n \geq c\sqrt{np(1-p)}$, which holds for **constant** n if $p > (1+\epsilon)k/n$.

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The End

Strategy for $p = 1/\max(|A|, |B|)$ when $|A \cap B| = 1$, $\mathcal{U} = A \cup B$, $|A| \geq |B|$

- Create bipartite graph whose vertices are all $|A|$-subsets (resp. all $|B|$-subsets) of $\mathcal{U}$ with an (X, Y) edge iff $|X \cap Y| = 1$. The $|A|$-subsets have degree $|A|$; the $|B|$-subsets have degree $|B|$.

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